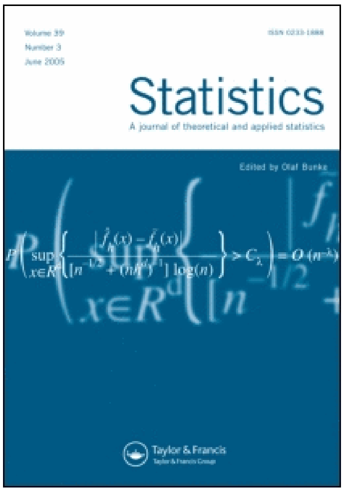

Statistics
Publication details, including instructions for authors and subscription information:
http://www.informaworld.com/smpp/title~content=t713682269

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To cite this Article Tiwari, Ram C. , Jammalamadaka, S. Rao and Zalkikar, Jyoti N.(1989) 'Testing an increasing failure rate average distribution with censored data', Statistics, 20: 2, 279 — 286
To link to this Article: DOI: 10.1080/02331888908802170
URL: http://dx.doi.org/10.1080/02331888908802170

statistics 20 (1989) 2, 279–286 Akademie-Verlag Berlin

Testing an Increasing Failure Rate Average Distribution with Censored Data

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Summary. A life distribution F is an increasing failure rate average (IFRA) if $\bar{F}(bx) \geqq \{\bar{F}(x)\}^b$, $0<b<1$, $x \geqq 0$, where $\bar{F} \equiv 1-F$. For testing H_0: F is exponential, versus H_1: F is IFRA, but not exponential based on randomly censored data, we propose a test statistic $J_n^c(b) = \int \bar{F}_n(bx)\, \mathrm{d}F_n(x)$, where F_n is the KAPLAN-MEIER product limit estimator of F. The asymptotic normality of $J_n^c(b)$ is established and an asymptotically distribution-free test is obtained. The efficiency loss due to censoring is studied compared to DESHPANDE'S (1983) test for uncensored case. The asymptotic relative efficiency with respect to CHEN, HOLLANDER and LANGBERG'S (1983) test is shown to be reasonably high.

Key words: Asymptotic relative efficiency, IFRA, asymptotic normality.

1. Introduction

A life distribution function (d.f.) F such that $F(x)=0$ for $x<0$, is an increasing failure rate average (IFRA) if $\left(-\frac{1}{t}\right) \log \bar{F}(t)$ is increasing in $t>0$, or equivalently if and only if, for $x \geqq 0$, $0<b<1$,

$$\bar{F}(bx) \geqq (\bar{F}(x))^b , \tag{1.1}$$

where $\bar{F}(x)=1-F(x)$ (see, e.g., BARLOW and PROSCHAN (1975), p. 84). The equality in (1.1) holds if and only if F is an exponential distribution. Let $X_1, X_2, \ldots, X_n$ be a random sample of size n from a continuous life d.f.F. For any fixed b, DESHPANDE (1983) proposed a statistic to test $H_0: F(x)=1-\exp\left\{-\frac{x}{\mu}\right\}$, $x \geqq 0$, $\mu>0$ (μ, unspecified) versus the alternative H_1 : F belongs to the IFRA class, but not exponential, by considering the parameter

$$M_b(F) = \int_0^\infty \bar{F}(bx)\, \mathrm{d}F(x) .$$

If G_n denotes the empirical distribution function, then $M_b(G_n)$ is equivalent to the U-statistic

$$J_n(b) = \frac{1}{n\,(n-1)} \sum_{1 \leqq i \neq j \leqq n} I\,(X_i > bX_j) ,$$

where $I(A)$ denotes the indicator function of the set A. DESHPANDE (1983) shows that the limiting distribution of $n^{1/2}\{J_n(b)-M_b(F)\}$ is $N(0,4\zeta_1)$, where under H_0

$$\zeta_1 = \frac{1}{4}\left\{1+\frac{b}{b+2}+\frac{1}{2b+1}+\frac{2\,(1-b)}{b+1}-\frac{2b}{b^2+b+1}-\frac{4}{(b+1)^2}\right\} \tag{1.2}$$

and computes the PITMAN's asymptotic relative efficiencies of $J_n(b)$ relative to the tests proposed by HOLLANDER and PROSCHAN (1972) and BICKEL and DOKSUM (1969) for three parametric families of distributions within the IFRA class.

In this paper, a test statistic is proposed to test H_0 versus H_1 with randomly censored data. Briefly the set up is as follows. Let $X_1, X_2, \ldots$ be independent identically distributed (i.i.d.) random variables (r.v.'s) having a common continuous life d.f.F. Let $Y_1, Y_2, \ldots$ be i.i.d. r.v.'s having a common continuous d.f.H, which is unknown and is treated as a nuisance parameter. Throughout, it is assumed that X's and Y's are mutually independent and the pairs $(X_1, Y_1), (X_2, Y_2), \ldots$ are defined on a common probability space $(\Omega, \beta, \mathsf{P})$. For $i=1, \ldots, n$, let $Z_i = \min(X_i, Y_i)$ and $\delta_i = I(X_i \leqq Y_i)$. Using the censored data (Z_i, δ_i), $i=1, 2, \ldots, n$, for a fixed b, a test is proposed to reject H_0 in favor of H_1 for large values of

$$J_n^c(b) = \int_0^\infty \bar{F}_n(bx)\, \mathrm{d}F_n(x) \,.$$

Here F_n is the KAPLAN-MEIER (KAPLAN and MEIER (1958)) product limit estimator of F defined by

$$\bar{F}_n(x) = 1 - F_n(x) = \prod_{\{i: Z_{(i)} \leqq x\}} \left\{\frac{n-i}{n-i+1}\right\}^{\delta(i)}, \tag{1.3}$$

where $Z_{(1)} < \ldots < Z_{(n)}$ denote the ordered Z's and $\delta_{(1)}, \ldots, \delta_{(n)}$ are the δ's corresponding to $Z_{(1)}, \ldots, Z_{(n)}$, respectively. The choice of b in $J_n^c(b)$ is discussed in Section 3.

For computational purposes $J_n^c(b)$ may be written as

$$J_n^c(b) = \sum_{j=1}^n \bar{F}_n(bZ_{(j)})\, \mathrm{d}F_n(Z_{(j)}) \,,$$

where

$$\mathrm{d}F_n(Z_{(j)}) = \bar{F}_n(Z_{(j-1)}) - \bar{F}_n(Z_{(j)}) \,.$$

In Section 2, the asymptotic normality of the sequence $n^{1/2}\{J_n^c(b) - M_b(F)\}$ is established under the following assumptions of CHEN, HOLLANDER and LANGBERG (abbreviated as CHL)(1983).

(A.1) The supports of F and H are equal to $[0, \infty)$,

and

(A.2) $\sup\{[\bar{F}(x)]^{1-\varepsilon}[\bar{H}(x)]^{-1}, x \in [0, \infty)\} < \infty$, for some $0 < \varepsilon < 1$.

Proposition (1.1) (GILL (1983)). *Assumptions (A.1) and (A.2) imply that the sequence of processes* $\{\xi_n(t) = n^{1/2}(\bar{F}_n(t) - \bar{F}(t)),\ t \in [0, \infty]\}$ *converges weakly to a* GAUSS*ian process with mean zero and covariance kernel given by* (2.1).

Note that the condition (A.2), as discussed in CHL (1983) restricts the amount of censoring allowed in the model. In the sequel, the test which rejects H_0 for large values of $J_n^c(b)$ is referred to as the IFRA test.

The null asymptotic mean of $J_n^c(b)$ is $\frac{1}{b+1}$, independent of the nuisance parameters μ and H. But, the null asymptotic variance of $J_n^c(b)$ depends on μ and H and must be estimated from the data. Using its consistent estimator $\hat{\sigma}_n^2$ an asymptotically distribution-free test is obtained which is also consistent against all continuous IFRA alternatives. In Section 3, a measure of the loss of efficiency due to censoring is derived using DESHPANDE's (1983) test and its generalization to the censored model proposed herein. It is shown that the IFRA test has fairly high efficiency when compared with CHL (1983) test. We also find the value of b which maximizes the efficacy (with respect to Weibull alternatives) of $J_n^c(b)$, for any given level of censoring. This section also contains an application of the IFRA test to some survival data and PITMAN's relative efficiency calculations.

2. Asymptotic normality and consistency

In this section, first the asymptotic normality of $J_n^c(b)$ is established. Also, it is shown that the IFRA test is consistent against all continuous IFRA alternatives under suitable regularity conditions. Our approach is parallel to that of CHL (1983). Let $\bar{K}(t)=\bar{F}(t)\,\bar{H}(t)$, $t\in(0,\infty)$, and let $\{\varphi(t),\ t\in[0,\infty]\}$ be a GAUSSian process with mean zero and covariance kernel given by

$$\mathsf{E}\{\varphi(t)\,\varphi(s)\}=\begin{cases}\bar{F}(t)\,\bar{F}(s)\int\limits_0^s[\bar{K}(z)\,\bar{F}(z)]^{-1}\,\mathrm{d}F(z), & 0\leqq s\leqq t<\infty\\ 0, & s<0 \quad\text{or}\quad t<0\,.\end{cases} \tag{2.1}$$

Unless otherwise specified, all limits are evaluated as $n\to\infty$, and all integrals range over $(0,\infty)$.

Theorem 2.1. *Let $0<b<1$. Under Assumptions (A.1) and (A.2) given in Section 1, $n^{1/2}(J_n^c(b)-M_b(F))$ converges in distribution to a normal r.v. with mean zero and variance σ_b^2 given by*

$$\sigma_b^2=\int\int\mathsf{E}\left[\varphi(bt)-\varphi\left(\frac{t}{b}\right)\right]\left[\varphi(bs)-\varphi\left(\frac{s}{b}\right)\right]\mathrm{d}F(t)\,\mathrm{d}F(s)\,. \tag{2.2}$$

Note that for $n=1,2,\ldots,$

$$n^{1/2}(J_n^c(b)-M_b(F))=B_{n,1}+B_{n,2}\,,$$

where

$$n^{-1/2}B_{n,1}=\int(\bar{F}_n(bx)-\bar{F}(bx))\,\mathrm{d}\bar{F}_n(x)-\int(\bar{F}_n(bx)-\bar{F}(bx))\,\mathrm{d}F(x)\,,$$

$$n^{-1/2}B_{n,2}=\int\left\{(\bar{F}_n(bx)-\bar{F}(bx))-\left(\bar{F}_n\left(\frac{x}{b}\right)-\bar{F}\left(\frac{x}{b}\right)\right)\right\}\mathrm{d}F(x)\,.$$

Hence, the proof of Theorem 2.1 follows from Lemmas 2.2 and 2.3 and an application of SLUTSKY's theorem.

Before stating Lemma 2.2, let us introduce some notation. Let $D=D[0,\infty]=\{\psi:\psi$ is real valued, bounded and right-continuous function defined on $(0,\infty)$, with finite left hand limits at each $t\in(0,\infty)$, and finite limits at $t=0,\infty\}$. Throughout D is viewed as a metric space with the SKOROHOD metric. Further, let Q^1, Q_b^2, Q_n^1 and $Q_{n,b}^2$ be the probability measures on D induced by the processes $\{\varphi(t), t\in[0, \infty]\}$, $\left\{\varphi(bt)-\varphi\left(\frac{t}{b}\right), t\in[0, \infty]\right\}$, $\{\xi_n(t), t\in[0, \infty]\}$ and $\left\{\xi_n'(t)=\xi_n(bt)-\xi_n\left(\frac{t}{b}\right), t\in[0, \infty]\right\}$ respectively.

Lemma 2.2. *Assume (A.1) and (A.2) hold. Then $B_{n,1}$ converges in probability to zero and $B_{n,2}$ converges in distribution to the r.v.* $\left(\varphi(bt)-\varphi\left(\frac{t}{b}\right)\right)\,\mathrm{d}F(t)$.

Proof. For $\psi\in D$, and $n=1, 2, \ldots$, let

$$\zeta_{n,1}(\psi)=\int \psi(bx)\,\mathrm{d}F_n(x)-\int \psi(bx)\,\mathrm{d}F(x)$$

and

$$\zeta(\psi)=\int\left[\psi(bx)-\psi\left(\frac{x}{b}\right)\right]\mathrm{d}F(x)\ .$$

By Proposition 1.1, Q_n^1 converges weakly to Q^1. This in turn implies that $Q_{n,b}^2$ converges weakly to Q_b^2. By the path-continuity of the process $\{\varphi(t), t\in[0, \infty]\}$ under (A.1), the support of Q^1 and Q_b^2 coincide with the set of all continuous functions in D. By the definitions of the mappings, we have $B_{n,1}=\zeta_{n,1}(\xi_n)$, $B_{n,2}=\zeta(\xi_n')$. Thus, to obtain the desired results, it is enough to show, by the extended continuous mapping theorem, that for every sequence $\psi_n\in D$ that converges to a continuous function $\psi\in D$, $\lim \zeta_{n,1}(\psi_n)=0$ w.p.1 and $\lim \zeta(\psi_n)=\zeta(\psi)$.

Now, recall that if $\psi_n\to\psi$ in D and ψ is continuous, then this convergence is uniform. (C.f. BILLINGSLEY (1968), p. 112.) Since $\lim \zeta_{n,1}(\psi)=0$ w.p.1, by simple integral evaluations we obtain that $\lim \zeta_{n,1}(\psi_n)=0$ w.p.1 and $\lim \zeta(\psi_n)=\zeta(\psi)$.

Lemma 2.3. *Assume (A.1) and (A.2) hold. Then $B_{n,2}$ converges in distribution to a normal r.v. with mean zero and variance σ_b^2 given by* (2.2).

Proof. From Lemma 2.2, $B_{n,2}$ converges in distribution to the r.v. $\int\left[\varphi(bt)-\varphi\left(\frac{t}{b}\right)\right]\mathrm{d}F(t)$, where $\left\{\varphi(bt)-\varphi\left(\frac{t}{b}\right), t\in([0, \infty]\right\}$ is a GAUSSian process. Also, under (A.2), it is easy to check that $\sigma_b^2<\infty$. Hence by the theory of Stochastic integration (see, e.g., PARZEN (1962), p. 78), $\int\left[\varphi(bt)-\varphi\left(\frac{t}{b}\right)\right]\mathrm{d}F(t)$ is normal r.v. with mean zero and variance σ_b^2.

To estimate the null asymptotic variance of $n^{1/2}J_n^c(b)$ from the data, let

$$f_b(z)=\frac{b}{(b+1)^2}\left[z^{2b+1}\left(1+b^2z^{2(b^2-1)}\right)-2bz^{b(b+2)}\right],\quad 0\leqq z\leqq 1,$$

$$\hat{\mu}_n=\left(\sum_{i=1}^n \delta_i\right)^{-1}\sum_{i=1}^n Z_i \quad\text{and}\quad \bar{K}_n(t)=\frac{1}{n}\sum_{i=1}^n I\,(Z_i>t)\ .$$

Further, let

$$\sigma^2(\vartheta) = \int_0^1 f_b(z)\,[\bar{K}(-\vartheta \ln z)]^{-1}\,dz, \quad \vartheta \in (0, \infty), \tag{2.3}$$

$$\sigma_n^2(\vartheta) = \int_0^1 f_b(z)\,[\bar{K}_n(-\vartheta \ln z)]^{-1}\, I(-\ln z < \vartheta^{-1} Z_{(n)})\,dz,$$

$$\vartheta \in (0, \infty), \quad n = 1, 2, \ldots,$$

and finally, let

$$\hat{\sigma}_n^2 = \sigma_n^2(b\hat{\mu}_n), \quad n = 1, 2, \ldots$$

Note, that under H_0, σ_b^2 given by (2.2) reduces to $\sigma^2(b\mu)$. In particular, when there is no censoring; that is when $\bar{H}(x) = 1$ for $x \geqq 0$, $\sigma^2(b\mu)$ reduces to $4\zeta_1$.

Theorem 2.4. *Assume $\sigma^2(\vartheta)$ is finite in the neighborhood of $b\mu$. Then under H_0, $\hat{\sigma}_n^2$ is a consistent estimator of σ_b^2.*

Theorem 2.5. *Assume (A.1) and (A.2) hold. Also assume that $\mu < \infty$ and that $\sigma^2(\hat{\vartheta}) < \infty$ in an interval that contains $\eta = \{\mathsf{P}(X_1 \leqq Y_1)\}^{-1}\,\mathsf{E}(Z_1)$. Then the approximate α-level test, which rejects H_0 in favour of H_1 if $n^{1/2}\left(J_n^c(b) - \frac{1}{b+1}\right)\hat{\sigma}_n^{-1} > z_\alpha$ is consistent against all continuous* IFRA *alternatives.*

The above results are similar to those of CHL (1983) and their proofs are omitted.

For computational purposes $\hat{\sigma}_n^2$ can be written as

$$\hat{\sigma}_n^2 = \frac{nb}{(b+1)^3}\left[\frac{(1-b)^2}{2n\,(b+1)} + \sum_{j=1}^{n-1} \frac{1}{(n-j)\,(n-j+1)}\left\{\frac{1}{2}\exp\left(-\frac{2\,(b+1)}{b_n\hat{\mu}} Z_{(j)}\right)\right.\right.$$
$$\left. + \frac{b}{2}\exp\left(-\frac{2\,(b+1)}{\hat{\mu}_n} Z_{(j)}\right) - \frac{2b}{b+1}\exp\left(-\frac{(b+1)^2}{b\hat{\mu}_n} Z_{(j)}\right)\right\}$$
$$-\frac{1}{2}\exp\left(-\frac{2\,(b+1)}{b\hat{\mu}_n} Z_{(n)}\right) - \frac{b}{2}\exp\left(-\frac{2\,(b+1)}{\hat{\mu}_n} Z_{(n)}\right)\frac{2b}{b+1}$$
$$\left. + \exp\left(-\frac{(b+1)^2}{b\hat{\mu}_n} Z_{(n)}\right)\right].$$

3. Efficiency loss due to censoring and ARE computations

Let F_ϑ be a parametric family within IFRA class with F_{ϑ_0} being exponential with scale parameter 1. (For example, one such family is Weibull $F_\vartheta(x) = 1 - \exp\{-x^\vartheta\}$, $\vartheta \geqq 1$, $x > 0$ and $\vartheta_0 = 1$). Consider the randomly censored model with $F = F_\vartheta$ and with censoring distribution H, and the sequence of alternatives $\vartheta_n = \vartheta_0 + \frac{a}{\sqrt{n}}$ with $a > 0$. Let $\beta_\alpha(J_n(b))$ denote the power of the approximate α-level tests based on $J_n(b)$ and n observations in the uncensored model and let $\beta_\alpha(J_n^c(b))$ denote the power of the approximate α-level test based on $J_n^c(b)$ and n observations from censored model. Let n' be a subsequence such that $\lim \beta_\alpha(J_n(b)) = \lim \beta_\alpha(J_{n'}^c(b))$, where the limiting value is strictly between α and 1, and let $k=\lim \frac{n}{n'}$. The value of $(1-k)$ can be taken as a measure of efficiency loss due to censoring. Since $J_n(b)$ and $J_n^c(b)$ have the same asymptotic means, k can be shown to be equal to the relative efficiency $e_H(J_n^c(b), J_n(b))=\frac{4\zeta_1}{\sigma^2(b)}$, where ζ_1 is given by (1.2) and $\sigma^2(b)$ is given by (2.3) which depends only on H and not on the parametric family F_ϑ of IFRA alternatives. When the censoring distribution is exponential, $H(x)=0$ for $x<0$ and $H(x)=1-\exp\{-\lambda x\}$ for $x\geqq 0$ with the restriction $\lambda<1$ (so that Assumption (A.2) is satisfied), the relative efficiency $e_H(J_n^c(b), J_n(b))$ becomes

$$e_H(J_n^c(b), J_n(b))=\frac{4\zeta_1}{b/(b+1)^2}\Big/\left\{\frac{1}{b\,(1-\lambda)+2}+\frac{b}{1-\lambda+2b}-\frac{2b}{b^2+b\,(1-\lambda)+1}\right\}.$$

The values of $e_H(J_n^c(b), J_n(b))$ for different values of b and λ are given in Table 1. As is to be expected, as λ tends to 0 (corresponding to the case of no censoring), $e_H(J_n^c(b), J_n(b))$ tends to 1.

Table 1
Asymptotic efficiency of $J_n^c(b)$ relative to $J_n(b)$ when H is exponential with scale parameter λ.

$b \backslash \lambda$	.9	.75	.5	.25	.1
.9	0.428	0.519	0.681	0.844	0.939
.5	0.454	0.548	0.707	0.860	0.945
.44	0.463	0.559	0.717	0.865	0.948
.25	0.508	0.612	0.764	0.890	0.955
.10	0.598	0.726	0.858	0.942	0.979

Again, let $\{F_{\vartheta_n}\}$ be a sequence of alternatives with $\vartheta_n=\vartheta_0+\frac{a}{\sqrt{n}}$, where a is an arbitrary positive constant and F_{ϑ_0} is exponential with scale parameter 1. From the results of CHL (1983) and Theorem 2.1, Pitman asymptotic relative efficiency (ARE) of the IFRA test with respect of the CHL (1983) test is given by

$$e_{F,H}(J_n^c(b), J_n^c)=\left\{\frac{M'(\vartheta_0)}{\Delta'(\vartheta_0)}\right\}^2\frac{\sigma^2(1)}{\sigma^2(b)},$$

where, $\sigma^2(b)$ and $\sigma^2(1)$ are null asymptotic variances of $n^{1/2}J_n^c(b)$ and $n^{1/2}J_n^c$ respectively, when H is exponential with scale parameter λ,

$$M(\vartheta)=\int \bar{F}_\vartheta(bx)\,\mathrm{d}F_\vartheta(x)\,,$$

$$\Delta(\vartheta)=\iint \bar{F}_\vartheta\,(x+y)\,\mathrm{d}F_\vartheta(x)\,\mathrm{d}F_\vartheta(y)\,,$$

are the asymptotic means of $J_n^c(b)$ and J_n^c respectively for the alternative F_ϑ and $M'(\vartheta_0)$ $(\Delta'(\vartheta_0))$ is the derivative of $M(\vartheta)$ $(\Delta(\vartheta))$ with respect to ϑ, evaluated at $\vartheta=\vartheta_0$. Note that $e_{F,H}(J_n^c(b), J_n^c)$ is the square of the ratio of the efficacies of $J_n^c(b)$ and J_n^c tests. Table 2 gives the values of b, say b-optimal, corresponding to the maximum PITMAN efficacy and efficiency of $J_n^c(b)$ test for Weibull family of distributions. This b-optimal corresponds to the largest local power, that is, it maximizes the probability of detecting H_1, when it holds. Also observe that as the amount of censoring increases, the optimal b-value in the above sense, decreases. This may be explained by the fact that the optimal test makes up for the loss due to censoring, by incorporating a larger number of observations from F, which corresponds to the smaller b-values. This is also reflected in the efficiency loss due to censoring, considered in Table 1. The ARE values in Table 2 indicate that the IFRA test has high efficiency when compared with CHL (1983) test, even for large values of λ (corresponding to the case of heavy censoring).

Table 2
The optimal b values for Weibull family of IFRA and NBU alternatives.

λ	Weibull family of IFRA alternatives	Weibull family of NBU-alternatives	maximum eff $(J_n^c(b))$	$e_{F,H}(J_n^c(b), J_n^c)$
0.0	0.44	0.5	1.166015	1.007104
0.1	0.35	0.33	1.136877	1.019619
0.2	0.29	0.33	1.108099	1.038152
0.3	0.24	0.25	1.079102	1.062527
0.4	0.21	0.20	1.049277	1.092463
0.5	0.19	0.20	1.017712	1.126682
0.6	0.16	0.16	0.983271	1.163767
0.7	0.15	0.14	0.944116	1.198534
0.8	0.14	0.14	0.897054	1.221266
0.9	0.14	0.14	0.837406	1.224559

The second column, corresponding to the NBU alternatives, optimizes the b values over the set $\left\{\frac{1}{k}, k=2, 3, \ldots\right\}$.

From the above table it is clear that in the absence of censoring, one may recommend $J_n^c(b)$-test with $b=0.5$ for NBU alternatives and $J_n^c(b)$-test with $b=0.44$ for IFRA alternatives. DESHPANDE (1983) recommends a b-value of 0.9 for IFRA alternatives. Although our computations do not justify that particular value, one may note that the efficiencies corresponding to $b=0.9$ and $b=0.44$ are not very different.

We now give an application of the IFRA test to some survival data. Table 2 of HOLLANDER and PROSCHAN (1979) contains an updated version of the data given by KOZIOL and GREEN (1976). This data has been analyzed by many methods (C.f. DOKSUM and YANDELL (1984)). The data corresponds to 211 state IV prostate cancer patients treated with estrogen in a Veterans Administration Cooperative Urological Research Group study. At the March 1977 closing date there were 90 patients who died of prostate cancer, 105 who died of other diseases, and 16 still alive. Those observations corresponding to deaths due to other causes and those corresponding to 16 survivors are treated as censored observations (losses). In (1.3) $Z_{(n)}$ is treated as death (whether or not it actually is) so that $\delta_{(n)}=1$. Furthermore, when censored observations are tied with uncensored observations, while ordering Z_i's, the convention is to treat uncensored members of the tie as preceding the censored members of the tie. For testing H_0 versus H_1 for the cancer data for $b=0.9$ we obtain $J_{211}^c(0.9)=0.533$ $\hat{\sigma}_{211}=0.030465$ and $(211)^{1/2}\left(J_{211}^c(.9)-\frac{1}{1.9}\right)\hat{\sigma}_{211}^{-1}=12.76$. The one-sided P value is 1.4×10^{-37}. Thus, there is strong evidence that an IFRA distribution is preferable to an exponential distribution. A plot of the estimated average failure rate does not disprove this conclusion.

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Received October 1986; revised May 1988.

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